Pontryagin Complexes

Theorem? A simplicial complex K is called a Pontryagin Complex iff its deleted join K*K is a pseudomanifold. It seems such complexes are necessarily joins of yin-yang complexes Y, i.e., simplicial complexes such that a set of vertices is a simplex of Y if and only if the complementary set of vertices is not a simplex of Y.

This result generalizes that of my 1980's paper *"Kuratowski Complexes"* in which the additional condition that K*K be 2dim(K)+1 dimensional was put.

Continuing this "note or a blurb if you will" -- see pp. 58-59 of this paper (my miscellany) let me say in pages preceding these there is much about the class of all yin-yang complexes; also about join factorization; join vs product surgeries etc. There is a pretty structure in this class which ties them to Milnor's definition of universal space for any group G as an infinte join. Indeed entire theory should go over from group Z_2 under consideration to any G.

The basic thrust is to turn back the clock as it were from characteristic cohomology classes to obstructing cycles, the claim being that *g e n e r i c a l l y* the presence of a nonzero characteristic class is equivalent to presence of the deleted join of at least one of a given finite list of Pontryagin Complexes.

The idea of genericity came in my 80's paper written right after, also while we were at the same Barrick Street address. This paper entitled *"Shifting and Embeddability"* I think of as the second in my Barrick Street Trilogy. The third, on its heels, attempted in vain to settle a problem using instead of Z_2 a bigger group G. But curiously of my papers in this field those falling under 'Barrick three' are most well-known, at least on-line.

This because Kalai fell imho excessively in love with my proof of Tverberg's Theorem, which has to an extent detracted from a proper assessment of my work. For example that an entire swathe of problems in this area were first raised in my pioneering omnibus paper "On neighbourly triangulations". As finale at the other end I looked at "Nice triangulations" -- see a later para after said page 59 of this miscellany -- that tie to Belyi's theorem.

Grothendieck also comes into yin-yang complexes-- in just a few pages of his melancholy Recoltes et Semailles. (In the 1970's when I was at Stony Brook it hit me really hard how cruel the 'business of mathematics' is that it had not even room to indulge and indeed l o v e someone who had given so much so unstintingly to mathematics!) He sticks though to the yin-yang complex RP^2_6 for which replacing all top simplices by their complements gives us an isomorph. Alas, I started musing, this *Icosahedral Theory* he starts is nowhere near as complete as it would have been if pursued by him in his heyday. The point being, this subcomplex of S^4_6 is the basic obstruction cycle for mod 2 characteristic class theory, and it is in this humble characteristic 2 that things are most subtle too. Maybe not just Arf theory, but the special wrinkle that of the symmetry groups only S_6 has *exotic* i.e. not inner automorphisms, has a role in this conjectural complete *IT?* I was about to enter such quixotic musings into my miscellany when fate intervened ...

For one certainly this conjectural IT should explain those (in the end very) fortuitous mistakes made first by Pontryagin and then years later by his student Rokhlin computing some homotopy groups of spheres. (Btw Rokhlin's student Gromov was the only person who really read my thesis before it was accepted.) There are mistakes it seems in all exciting mathematics. Almost all work of Riemann, Poincare and Lefschetz would not survive a mechanical line-by-line checking for accuracy. And these above are five of the names from the very heart of mathematics! Setting aside false modesty let me say again mistake-prone K S Sarkaria has also broken some totally new ground!

I had always been interested in, and frustrated by, not understanding physics. Realizing finally I ought to stop castigating myself, and junk empty "laws" like F = ma -- this is not in Newton, btw to me the way proofs are written in his Principia remains the ideal way: as few mathematical symbols as possible! -- and not accept matter or energy as fundamental. Basic are only space and motion. Matter can be then shown to exist as various forms of motion. Going back to this idea of Descartes we make (this more general cartesian) physics a part of mathematics itself, from which all its other parts will get stimulation. (Of course of this huge cartesian physics what is in conformity with what say a linear accelerator stirs up is for "those" physicists to decide. That is a market place, so complicated are the experiments of this kind I wager even good theoretical physicists are at sea about them. A consensus is finally built in a complicated sociological way ...). This long pending project of mine finally fructified in a nice way with my *five-part paper "Plain Geometry and Relativity"*. (Btw if you keep ignoring my papers that do not look like journal papers you'll never understand me.) It turns out indeed that every closed manifold occurs thus as a form of motion only. And *if we accept commonsense and junk infinite extent* – (so easy a point, I put it even in a historical satire, A Turtle's Tale : completing this pic with text, and before this, two pending transations is all I've done since; this blurb too is being written without actually checking my papers... as I wrote on that p.59 it's going to be a long haul) -- *then we have relativity* ... With this commonsense accepted *matter coming from this relativistic motion automatically has a Lipschitz structure*. Sullivan's argument shows in dimensions other than 4 a little shaking of the motion gives any manifold a unique Lipschitz structure. In dimension 4 only there is obstruction : every manifold is the image of a continuous map from the interval, i.e. is non-relativistic, but not all are born of a relativistic motion. This subtle mod 2 obstruction too should be clearly explained by IT ...

By making absolutely natural demands one can show how cobordism of these manifolds (matter) occur. You'll have to read PG&R 1-5 for more details. To some the glass would seem half full, to others not even that, but surely the *e v i d e n c e* that I give for the theorems claimed will be found at least stimulating.

One of my spurs for writing up this cartesian creation of matter was Gromov's paper : "Manifolds, what are we, where do we come from etc." Which I found a bit heavy-handed and ad hoc. Otoh the basic *cartesian credo : matter is but forms of motion : gave a very natural genesis and evolution of manifolds*.

For sure IT should explain clearly what all is in that *fascinating paper of Riemann*, which, spurred by Atiyah, Mumford did put some time into understanding; but *in my opinion Mumford made a meal of it!* What Riemann can see at once, in the Tata Theta book, if at all there, it is buried in formulas. A proper exegesis of Riemann by a worthy mathematician, which sticks close to the s p i r i t of the original -- like mine of Poincare's Analysis Situs -- is still lacking vis a vis Riemann.

Not only Ruffini and Abel but Galois too thought quite topologically. And with that sweeping theorem on page 380 of Jordan's book, it is clear *this is the heart of and the way to think about Galois theory.* Alas the clickety-clack group theoretic parts *only* of Jordan were imbibed, modern books in Galois theory are but that.

The fact known to Betti that insolubility of a generic equation of degree greater than four by radicals is a purely topological result passed on from him to his friend Riemann, who worked out that indeed this topological way also tells us how to solve the generic equation. This is what that theorem on page 380 of Jordan does.

The five-part (or six-part if you count part 3.5 too) *paper "The prettiest composition (of Khayyam)"* begins by drawing attention to someone who really had pushed algebra beyond what is in Euclid. *It is funny that the school qaida of Khowarazmi which penetrated Italy and set Europe down the terrible path of* *notation, notation and more notation, is held to be the sourcebook of algebra!* There is a natural stratification of the space of sequences of real numbers if one considers them as equations and looks at subsets where the number of real roots is a constant. These swallowtails, generalizing a terminology of Thom, are very beautiful and secondly they do fit naturally in the theory presented in PG&R. Going to complex numbers washes all this natural beauty but of course through Hodge theory offers a practical way of producing some results. In particular Adiprasito using this technology got the Heawood Inequality conjectured by me, and thus, as had been known for long, much about the numerics of some triangulated manifolds. But *naturality demands we stick to reals and see our way through using these only*. Confining myself to the generic part of the swallowtail I was able to directly see how *to solve equation of any degree by using just some trigonometry from Hobson's book. This Hobson's choice as I call it in fun,* is undoubtedly linked to Riemann's higher dimensional theta functions, also defined in terms of trigonometry. With the later discovery by Poincare of automorphic functions this solution should be put explicitly and beautifully in terms of these: but it has not been done. As an existence statement though it is a consequence of the uniformization theorem. I wasn't able to get to the bottom of the well, but mostly only because of *my wiring* -- maybe my curious bio holds the clue to this? -- *which insists everything be both simple and beautiful* always. So precluding use of off the shelf results, or techniques that I deem do not satisfy the cartesian criterion of being totally natural and clear.

Most recently, working in the middle of items I had already written in my miscellany, I realized genericity restriction is just the peg using which we can go from characteristic classes to a finite list of obstructing equivariant cycles. *Once again this all was just a blurb* and nothing more: for the real stuff you'll need to study my papers mentioned above.

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