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Some minimal nonembeddable complexes

(This came to me from MR; in Bonn, Schild who came with Böhme had given me a preprint). (My review is on back page)

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Abstract

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The purpose of this paper is to give a family of simplicial complexes which are not embeddable in a linear space of fixed dimension. Additionally it is shown that "almost all" of these complexes are minimal with regard to this property.

Keywords: Simplicial complex; Embedding.

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1. Introduction

It is well known that every n-dimensional simplicial complex can be linearly embedded in R^{2n+1}. However, there exist n-dimensional complexes that are not (topological) embeddable in R^{2n}. In [1,2,11], two classes of such complexes are given. These results were generalized in [4] using the concept of the join of complexes. In [4,14] it was also shown that the considered complexes are minimal, i.e., every proper subcomplex is linearly embeddable in the appropriate space. Continuing the investigations of [8] we present a class of complexes, including all those considered in [1,4,7,11,14], whose members are minimal in the above sense. In contradistinction from the above papers we do not only examine the embeddability of n-complexes in R^{2n} but also in R^q where $q \neq 2n$.

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In this elegant paper a simplicial complex K is called *nice* iff, for each $\sigma \subseteq \mathrm{vert}(K)$, either σ or σ^c, but not both, are in K; also a closed simplex is considered "nice" by convention. It is shown that, if a simplicial complex L is a join of r such nice K_i's, each having $n_i + 3$ vertices, then L is *not embeddable in $\mathbb{R}^n$*, $n = n_1 + n_2 + \dots + n_r + 2r - 2$. Furthermore, barring obvious exceptions, all such L's are *minimal* with respect to this property, since all their proper subcomplexes do embed in $\mathbb{R}^n$.

Reviewer's Note: The non-embeddability also follows by using the Borsuk-Ulam Theorem, and the fact that the *deleted join* L_* (see reviewer's cited paper for definition) is an an antipodal $(n+1)$-sphere. We note also that the six-vertex real projective plane and the nine-vertex complex projective plane are "nice".