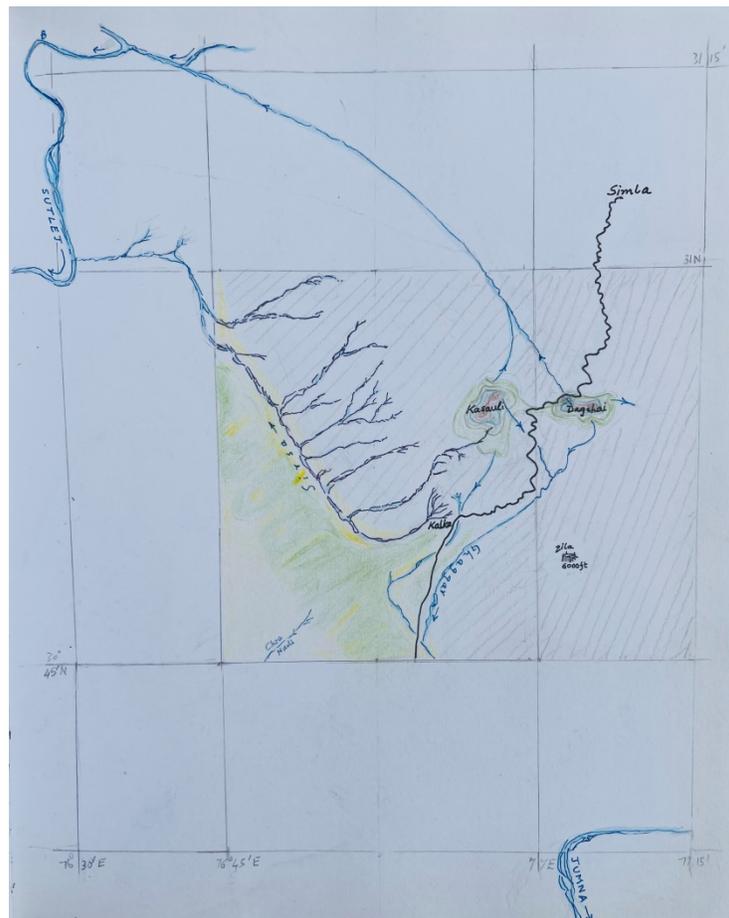


Sirsa nadi

We all know how the Guru's family was separated for ever in the swollen Sirsa near its confluence with the river Sutlej. These events of the early 1700s are solemnly remembered every year in a very big way. With some even wading at midnight through waters representing this nadi in a few gurdwaras.

Nevertheless when one searches for this nadi the results that pop up are very disappointing. At most just the final portion of this stream is drawn, and on current maps of Punjab usually no nadi is named Sirsa. Our historical records are in a bad way, but here we are talking of our geography. Surely I thought the nadi still flows, albeit a bit differently after all these years, and it should be possible to trace it back upstream to all its sources. Thanks mainly to maps A, S from an on-line library of University of Texas, Austin, the answer was soon found. Since these maps are very detailed I'll use a sketch based on them to spell it out:-



The sources of the Sirsa are almost all in the hilly hatched area of the central rectangle of the sketch, i.e., between longitudes $76^{\circ} 45'$ E and 77° E and latitudes $30^{\circ} 45'$ N and 31° N, i.e., in one of the 24 rectangles—viz., the second from left in the top row of 6 rectangles—with maximum difference of latitude or longitude 15 minutes, into which map **A** can be subdivided.

In the like rectangle to its right—drawn in the sketch with horizontal scale halved—there is marked in map **A** a Jagatgarh Fort and we learn that this qila was then in P.E.P.S.U. However the photos to which I've just linked were taken much later in 2007, and it was likely soon after this trip to Banasar Garhi that I found this nice map library, and in particular map **A** on the web.

In fact I've drawn in the sketch around the central rectangle a ring of eight like rectangles with the horizontal or the vertical or both scales of each halved. Of these nine rectangles the three at the top belong to map **S**, viz., are the first three rectangles in the bottom row of six if we subdivide map **S** into 24 rectangles with maximum difference of latitude or longitude $15'$.

I note we can put around these 9 a new ring of 16 like rectangles with the horizontal or the vertical or both—this for the 4 corner rectangles—scales halved again, and so on. A simple rescaling program that allows us to see terrain depicted over any number of such rectangles in a cohesive way within a rectangle only three times bigger than any one of them that we want to focus on.

Returning to our focus, you'll see winding through this central rectangle and two others the Ambala to Simla road, starting from just a little south of the Mughal gardens in Pinjore, which lie between the two tributaries of the Ghaggar drawn in the sketch. These and other tributaries that I've not drawn of this river also arise in the same hills, but whereas Ghaggar dies in the Thar desert, the waters of the Sirsa via Sutlej and Indus are headed for the Arabian Sea. Indeed as depicted following map **A** there is a bifurcation of waters on the southern slopes of the hill looming over Kalka itself : some drain into the southernmost tributary of the Sirsa, the rest into the northernmost tributary of the Ghaggar.

In that hatched hilly area there are many massifs rising above 6000 feet of which I've highlighted only two, Kasauli and Dagshai, by drawing around them some mostly decorative contours, i.e., curves showing equal height. As against this

maps **A**, **S**, etc., are all very finely and fully contoured, with a host of springs and nallahs shown running down lines of steepest descent.

From all this topography in **A** it is clear that Kasauli's western and most of the higher southern slopes contribute to the second tributary of the Sirsa: the tendril that I've chosen to draw in the sketch being a spring that still runs below village Jangeshu. However across a ridge to its east there is a nallah which avoiding the Kalka-Parwanoo obstruction slips under the road to fall in the other tributary of the Ghaggar included in the sketch. As for the waters down the northern slopes of Kasauli-Sanawar some reach Sabathu doon in the north-eastern corner of the central rectangle, joining nadis flowing north-west that fall too in Sutlej, but well upstream of Sirsa, indeed even before a deep narrow gorge **B** near village Bhakra, through which the Sutlej used to flow freely when **A** was drawn.

Fine and full contouring gives us height $h(\theta, \varphi)$ at all latitudes and longitudes. Assuming other factors uniform, which paths of steepest descent should be deemed rivers? Clearly not all such paths should be: for example if h is smooth enough then through any point, other than those where $-\text{grad}(h)$ vanishes, there exists a unique path of steepest descent, viz., the integral curve of this vector field through that point, but we certainly wouldn't like to dub all these curves rivers. The key to a reasonable definition is the realisation that a river is not a one but a two dimensional subset of the map. For any element of area α if we produce the integral curves through it all the way back we get its catchment area β . A river is composed of α 's for which the ratio β/α 'blows up' but if this ratio is not that huge but still big they constitute a nadi, etc.

Higher doesn't necessarily mean farther from the centre of the earth, e.g., Mt Everest is closer to it than many places near the equator. The longitudes are not circular: if we cut the earth by a plane through its axis of rotation we get an ellipse with this its minor axis, the major axis is about 26 miles longer. This ellipsoid of revolution nature of the earth matters for regions as big as in **A**, **S**, etc., and has duly been taken into account by their careful cartographers.

Turning to Dagshai the run-off of its northern slopes—there is such a nallah a stone's throw from Singuffa—hurtles down during the rains to Sabathu valley to join rivers Dabar-Gambhar that fall in the Sutlej before Bhakra. On the other hand

its southern slopes drain into Kiarad doon—in which I’ve tramped with Manpreet and Mallika, and across which high on Banasar is that qila—and these waters are headed for the Ghaggar. On the far eastern end of the Dagshai massif after Mem di Kabar between village Nahech and Kumarhatti are nallahs which may be headed all the way to the river Jumna—also depicted in map **A**—so via the Ganges across this sub-continent for the Bay of Bengal.

The Sirsa nadi is constrained to its north-western course in the central rectangle by a range of low hills along its left bank. The map **A** shows these hills drain towards the plains into many a nadi, rao or choa—all three mean stream—all roughly perpendicular to this range. I’ve drawn only one in the sketch because of its unassuming name: choa nadi. Also because, around that south-western corner of this rectangle was built later a city, and this ‘stream stream’ goes through a rose garden a stone’s throw away from binguffa!

Notes

1. A popular date for the separation in the Sirsa is December 20-21, 1704. In a primary source this date would likely be in Bikrami. Which reminds me of another item in my in-tray: to write a full concise definition of this calendar – no such definition seems to exist – using which and paper and pencil any highschool kid can convert dates from and to Bikrami. Indeed some months ago I chanced upon a 19th century tome in which definitions of some Bikrami calendars - including it seems the one important for Punjab’s history - are given, but in a long-winded and tabular form. Hopefully a pithy definition can be given after perusing this book, but so far I’ve not found the time to do this.

2. Now that I’ve taught myself to type Punjabi in shahmukhi maybe I can myself begin to decipher the mss attached to The Forgotten Koh-i-Noors of Ranjit Singh (2014) as follows: type what characters I can make out—Farsi alphabet is a subset of Urdu alphabet but it is likely these salary records also contain characters of an accounting language called siyakat—and then try machine translation from Farsi. Once again: the entire official records of the fifty odd years of the Khalsa Raj are conveniently available but our “historians” can’t be bothered about reading them.

3. There is no distance preserving map of even a small region of an ellipsoid of revolution in the euclidean plane, because the gaussian curvature of the former is positive at all points, while that of the latter is identically zero. However if from our map we can read the longitude and latitude of any two points, we can compute the distance between them to any desired accuracy, and it hardly matters for this purpose that if the ellipsoid is not a sphere the closed formula for the distance – see note 11 below for shortest paths – involves an elliptic integral, but no virginia! the circumference of any ellipse with semi-axes a and b is not $\pi(a+b)$ even though the enclosed area is $\pi(ab)$.

4. That that qila was in Patiala and East Punjab States Union when **A** was drawn almost confirms that it became part of Patiala state once the forces led by Ochterlony—the biggest contingent Sikhs from Patiala which was now ruled by Karam Singh—had prevailed upon Amar Singh Thapa and his men in the

Anglo-Gurkha war of 1814-1815. There are available blow-by-blow accounts of some battles written by Britishers—this qila does not seem to figure in any—and names of all firangis who fell are still preserved, but almost nothing is known about the vicissitudes of the patialvis in this war. This despite the fact that Patiala was a big winner: it doubled in size after this victory. This state of affairs may be because our “historians” have not bothered to read the Patiala state archives either. The name Jagatgarh Fort given to this qila in map **A** may or may not be tied to the fact that one Jagat Prakash was once a ruler of Sirmaur state which had ended up being on the losing side of this war.¹

5. Likewise if the reduction of scale is by a factor r in each successive ring of rectangles then any number of rectangles can be put in an open rectangle U of size $(1+r)/(1-r)$ times the central: because the sum of a geometric series with first term 1 and common ratio $r < 1$ is $1/(1-r)$. The ‘straight lines’ of this map-reading geometry are usually not straight, only piecewise straight, however it satisfies the parallel postulate: indeed these rescalings just transfer the geometry of R^2 tiled by equal rectangular tiles to U , and the symmetries of the ensuing regular tiling of U allow us to choose any tile as central. More generally there is a similar homeomorphism of euclidean n -space R^n equipped with an n -cuboidal tiling onto an n -cuboid U of size $(1+r)/(1-r)$ times any tile.

6. We can use too for map-reading the cayley geometry of the open rectangle U —now geodesics are straight and the parallel postulate is false—for it admits an almost regular $\{4,4\}$ tiling. As against this the more homogenous cayley geometry of an open disk B does not admit such a tiling. Further all this generalizes to higher dimensions but for $n > 5$ it is hard to show even that an open n -ball B with the cayley metric—or equivalently the hyperbolic metric: the two are related by a radial homeomorphism identity on the boundary of the ball—admits any regular tiling. On the other hand an open n -cube U with its cayley geometry clearly admits an almost regular tiling with quotient an n -torus. It seems that using a map-reading geometry an argument of Sullivan, showing that for $n \neq 4$ any n -manifold has a unique lipschitz structure, can be dramatically simplified.

7. There are also more detailed maps of the 4×6 rectangles into which the maps **A**, **S**, etc., can be subdivided, but these are not open access. Luckily I had obtained in 1994 or so maps **A**₁₂ and **A**₁₃ of the two main rectangles of the sketch: for a planned day hike to and back from the qila. Indeed there is in **A**₁₃ an unnamed oblong – a forest near it is named Jagatgarh R F – at height 1569 m and from the legend at bottom-right we learn that this is an inhabited deserted fort whatever that may mean: possibly that it had been deserted by its owners but some others were living in it ? Anyway it was likely after that 2007 trip – when we found the qila deserted and uninhabited – that I found the earlier map **A** with a Jagatgarh Fort marked in it: does this perchance mean that it was then occupied by its patialvi owners ? About the Sirsa there is much more information in **A**₁₂: even names for the tributaries of this ‘forgotten’ nadi ! Also in the south-west corner of this rectangle a budding city has now supplanted some villages. But the Pinjore and Kalka bypass, which rejoins the old road to Simla after Parwanoo is in the future; and the horrific four-laning of this old road from here on by some “engineers” still further in the future: so badly scarred are these hills by it that they shall take years to heal.

¹ July 19, 2023. I’ve now attached also three older maps from the same library: from nahan 1916 we learn not only that Banasar tower or garhi was indeed in Patiala state, but also that so were two more towers atop this hill. However it would be a bit trying for me to go hiking now to bring back for you photos of what, if anything, remains by way of ruins of these two fallen towers.

8. Also I used maps **J** and **L**, from the same nice library in Austin, to follow the Sutlej to the west of **A** and **S**. The eponymous tributary of a river is usually but not always that which contributes the most flow, or which makes it the longest. For the Sutlej it is that which flowing westwards right across **S** met roaring nadis from the north and the south in the top left of the sketch. This geography is now history : thanks to the Bhakra dam all this and even the old seat of the Bilaspur raja is submerged under the Gobindsagar lake. It would be nice to see even older editions of these great maps—which reminds me: if one takes a dandi, from the first hawaghar on the road up to Dagshai cantonment, down towards Kiarad valley, there is at a stone's throw, but slightly off to the right on a hillock, a survey marker which might date back to when the Great Trigonometrical Survey of India reached these cis-Sutlej hills—for these older maps should almost tell us the geography of this area as it was in the early 1700s : man has wrought almost all changes on nature only after 1900 or so.

9. It suffices for the existence and uniqueness of an integral curve of $-\text{grad}(h)$ through any point that this vector field be lipschitz. However it is more realistic—e.g., those 'trees' of tiny rivulets as we follow the Sirsa upstream to its sources—to take $\text{grad}(h)$ only continuous: now we have existence, but not uniqueness of a curve of steepest descent through each point. Also there are sharp ridges or critical points where h is not even continuously differentiable: so there are ifs and buts to my notion of a geometric river. However much about the topology of n -manifolds has been found mulling the contours of a generic smooth function, e.g., Milnor saw that a smooth 7-manifold $T_1(S^4)$ not diffeomorphic to S^7 admits such a function with two critical points, so is homeomorphic to S^7 . As regards the third dimension of streams, things haven't progressed much beyond divining rods when it comes to finding and tracing the flow of subterranean water. Which reminds me: in the qila is a defunct bauli, which was probably the reason why this particular site was chosen for it on this dry hill.

10. The charts **A**, **S**, etc. use a projection due to Gauss : a suitably scaled, to make it conformal, projection from the centre of the earth on a cylinder wrapped not around the equator, as in Mercator's projection, but around some chosen meridian: for these maps through Kalyanpur in central India. In this transverse Mercator projection line segments are in a fixed direction with respect to east, but by no means are they shortest paths. Assuming the earth an S^2 its geodesics are great circles, i.e., its sections by planes through the centre of the earth. So were we to convert—maybe by some app?—these maps to projection from the centre on the tangent plane at this village we could go Straight to Mecca from any point in this subcontinent. Anyway in the linked paper I used central projection to prove a theorem of Gleason, which is verily the lynchpin of a cottage industry called often quantum logic.

11. The ellipsoid of revolution nature of the earth perturbs shortest paths in an interesting way: the only sections by planes through the centre which are still geodesics are the equator and the elliptical meridians. The geodesic starting from a point P on the equator, in a direction between east and north, now does not meet the equator next in $-P$, but in a point Q of a small open equatorial arc before it, and this northerly geodesic arc from P to Q is shorter than the equatorial. The symmetry of the problem shows that the geodesic continuing this arc shall swing around the equator a number of times, maybe even infinitely many times—this iff the equatorial arc PQ is an irrational fraction of the equator—and cut itself again and again, before it returns to P , if ever. The exact calculation of that 'open equatorial arc' and these ellipsoidal geodesics involves abelian integrals and was understood only when Jacobi finally managed to extend it to any ellipsoid whatsoever.

12. In that nice library, besides the series of maps to which **A, J, L** and **S** belong, there is a later 1982 series whose cartography is different: its relief shading makes contours easier on the eye, but many place names are omitted. From the map **A+** of this series it seems even more likely that during heavy rains water down a nallah near Nahech finds its way to the Jumna. On an entertaining drop-a-drop-and-see-it-flow app, though there was no provision to drop exactly at this latitude and longitude, from Solan a bit to the east the drop did flow—at rocket speed!—via Jumna, Gunga to the Bay of Bengal. However to understand terrain you have to hold it still, and read a map like a book. I found **A+** helpful in many other ways too, for example, since ugly highways are in the future, that 35 km loop of almost daily road cycling I used to do is depicted in it almost exactly the way it was.

13. There seems no end to what reading these maps reveals: that choa nadi = stream stream which “flows” through the rose garden in sector 16 is shown even in the 1982 map to go past Rajpura and feed near Ambala into the beginnings of the Narwana branch canal! I wrote “flows” because now there is hardly ever a drop of water in it. But yes, about 10 years ago during a heavy downpour a youth got swept away in this garden by this nadi, with his body discovered later only in sector 23; and it is in flood again right now because of heavy rain even as I write this. With rapid denudation, pollution, uncontrolled construction – to which one might add no concern even by the very devout for the present state of a nadi which figures so high in our tradition – our geography is changing very rapidly indeed.

K S Sarkaria