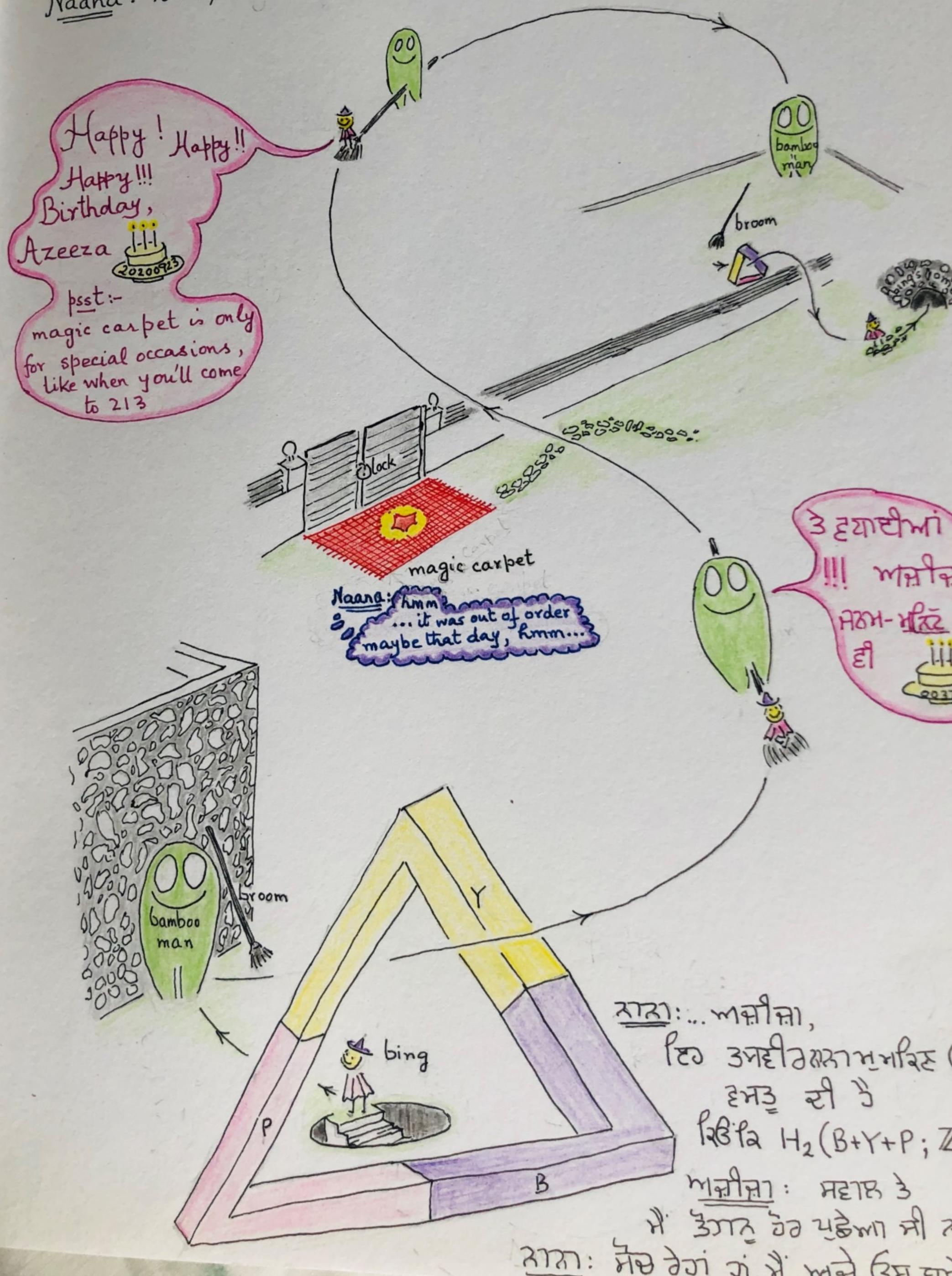
bing, broom and bamboo man

Naana: here's how bing flew bamboo man out on broom I think...
Azeeza: but why didn't he use magic carpet? That's way more comfortable!
Naana: hmm, hmm, ...



ਨਾਨਾ: ... ਅਜ਼ੀਜ਼ਾ,
ਇਹ ਤਸਵੀਰ ਨਾਮੁਮਕਿਨ (!)
ਵਸਤੂ ਦੀ ਹੈ
ਕਿਉਂਕਿ $H_2(B+Y+P;\mathbb{Z}) \neq 0$
ਅਜ਼ੀਜ਼ਾ: ਸਵਾਲ ਤੇ
ਮੈਂ ਤੁਹਾਨੂੰ ਹੋਰ ਪੁੱਛਿਆ ਸੀ ਨਾਨਾ।
ਨਾਨਾ: ਸੋਚ ਰਿਹਾਂ ਹਾਂ ਮੈਂ ਅਜੇ ਉਸ ਬਾਰੇ, ਬੇਟੀ।

You'll enjoy the picture story on the previous page more if you first read Cacti and Mathematics (2007), which was written for a lay audience, and at least some pages from "213, 16A" and Mathematics (2010), which, despite a similar appearance---a near absence of symbols that to many are the sine qua non of my muse, and a profusion of photos and drawings---is a challenging maths paper, especially for a beginner: mastering its 37 pages will take a talented high school student to post-graduate level and beyond!

However its pages 12-14 should be easy enough to absorb, because there I am only talking about--as against doing--mathematics, but are important, for they give you in a nutshell my mathematical philosophy and methodology, and you can rest assured that I myself try to work just as hard and (nota bene) 'seriously' as you'll find me there exhorting others to do.

One of the dramatis personae in the picture story appeared (or did he?) before in **bing's ome**, so you should read also this story on its concluding pages 33-37. Besides what any story should be, fun, it has quick proofs that a certain house with two rooms is contractible but not collapsible, and that all second homotopy groups are abelian. But a beginner can skip these, and still get fully the other important thing, a moral, which any good story has. This surprisingly being that the paper, all along apparently about some architectural motifs, was really not about them, but about some evoked perfect mental images that we can never hope to depict precisely: The Elves of Mathematics.[1]

Playing a major rôle in the pictured story is also **bamboo man**, a not always as-manicured-as-it-should-be topiary, but seen from afar it has a surface--note those through-and-through holes it has for its eyes--of genus two. This here being not the genus of the living species to which bamboo man belongs, but the genus of the evoked mathematical elf, surface, which is so fussily defined that we can never hope to depict it precisely. Of course we mathematicians know the upside of this fussiness too, but let me not go into that here.

[1] Accordingly, when C S Aravinda asked me in 2019 if Bhāvanā could re-publish this paper, I said yes, but that its title should be changed to this, and it has since appeared under this title in two parts in the January 2020 and April 2020 issues of this magazine: I am very thankful to him for further popularizing the elves!

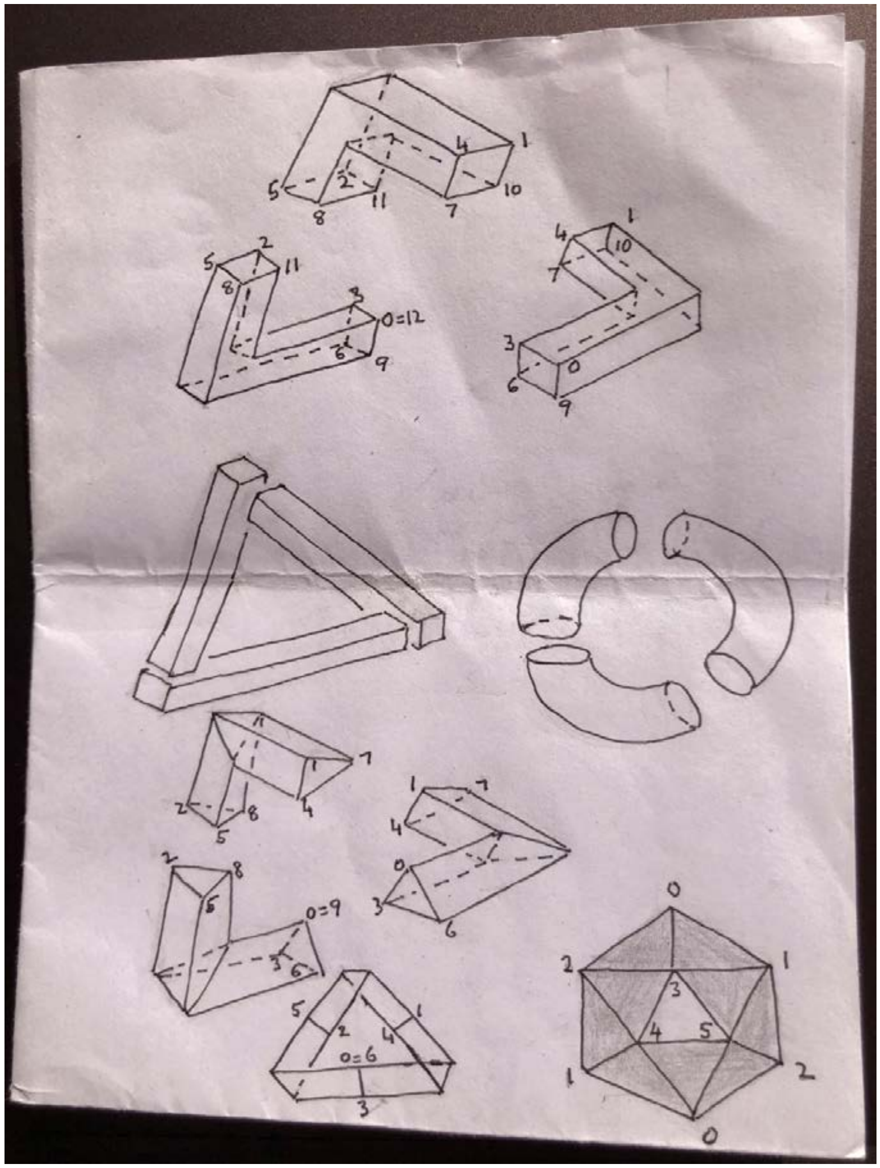
Using for cactus lovers this definition of a surface, as the skin of a physical object, that earlier and easier paper ties patterns on plants to quite a bit of beautiful mathematics, which was my main object, but en passant, a minor botanical mystery was also resolved in its dénouement!

Coming to the action in the picture, it all began when one morning I saw bamboo man standing far away from where he had been just the evening before! Recovering, and refocussing my faculties, I saw then that the **broom** which had been near it the day before, was now near its new position: putting two and two together I surmised that overnight what I've drawn must have transpired. I was a little apprehensive about my depiction of bing though, for, despite repeated efforts over the years, frankly I have still not spotted this elf.

As you can see this picture story was drawn for my grand-daughter Azeeza's birthday, and I'm happy to report she liked it; but, from the to and fro it is clear I had then no convincing reply to the poser as to why bamboo man had to go riding a broom when bing could have used that comfy **magic carpet** which was equally handy? Let me recall here for the more mathematical that it indeed is magical: in the eighth and final "lecture" of that 2010 paper is a full proof of a beautiful, profound and sweeping theorem suggested by this motif!

So, turning the conversation back to what else I had drawn, you can read me telling her, about an object well-known to be 'impossible' or ਨਾਮੁਮਕਿਣ, that it was actually ਨਾਨਾਮੁਮਕਿਣ ਵਸਤੂ, i.e., an **imimpossible object**, because - ahem! - the second integral homology group of it's surface is nonzero.

The cognoscenti will see what I just said is equivalent to saying that a closed and connected surface S is orientable or two-sided, and that objects bounded by these are exactly all those we can ever hope to see in 3-space, the two sides being towards which the material of the object is, is not. To clinch matters one of the doodles below shows how, because the three angular ells into which I had cut that ਵਸਤੂ are topologically curved cylinders, they do form after pasting ends a solid torus in space. Also homology is computable, so the *characterization* $H_2(S;Z) = 0$ *of a topologically impossible surface* S is often helpful.



Another doodle explains the name **tribar** for our vastu, but it has been cut into three ells in the doodle at the top. The numbering in it indicates a long closed curve which cuts any transverse section 4 times whereas there is only 1 cut made by the locus of centres of sections. This non-triviality of a bundle, equivalently of a characteristic class, defined by the putative cell structure is why, *the tribar is impossible to find cell-wise flat in 3-space*. Another doodle indicates the same obstruction if the cross sections were 3-gons, likewise for p-gons; even for the seminal case of 2-gons or segments for at least one more wrinkle or edge appears when we ‘make’ *the mobius strip through which bing jumps to return ome in the picture story!* But, as another doodle – “Beesmukhi” seen from the top! – shows, there is a 6-vertex simplicial mobius strip bounded by just a triangle. Obstruction to linear non-embeddablity in simplicial n-complexes topologically embeddable in m-space was what my 1992 paper with Brehm was about. Also, an obstructing cohomology class may be said to belong to a doodle suggesting that cell structure, the context in which it was discussed in 1991 by Penrose.

Coming back to that query of Azeeza's which has me stumped in the picture story I later saw the reason why bing opted for a broom : its longueur allows us to see clearly the bias of a right handed screw, first up and then down, in the flight path bing followed! The same uncanny bias seen in the strands of all life, DNA, whether it be in those like bamboo man converting carbon dioxide and water to sugar and oxygen, or in those like me converting them back to carbon dioxide and water. This suggests, *not only are all surfaces that are found in space orientable, they come with a natural orientation dictated by life itself!* Note further the arrow of time on the flight trajectory. Time in all life is the sensation of ageing, the dying off and the birth of new strands, which get sequenced in one of their two total orders. The natural orientation of space, i.e., the space of all right handed screws, is also double covered by the connected space of all directed right handed screws, which suggests, *all surfaces of objects in space come with a natural spin structure dictated by the double helices of life throbbing within them!*

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K S Sarkaria